**Laboratory work 6**

**Analysis of accuracy decrease of filtration in conditions   
of correlated biased state and measurement noise**

Performance -Thursday, April 15, 2016

Due to submit a performance report – Wednesday, April 20, 2016

The objective of this laboratory work is to analyze the ensitivity of estimation results obtained by a Kalman filter that doesn’t take into account bias and correlation of state and measurement noise. This will bring about a deeper understanding of main difficulties of practical Kalman filter implementation and skills to overcome these difficulties to get optimal assimilation output.

The first part of this laboratory work is performed in the class by students as in teams of 2 on April 15, 2016 and the second part on April 19, 2016, and the team will submit one document reporting about the performance till Wednesday, April 20, 2016. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

This laboratory work consists of two parts:

1. Divergence of Kalman filter when bias of state noise is neglected in assimilation algorithm. Development of optimal Kalman filter that takes into account bias of state noise
2. Sensitivity of estimation results obtained by a Kalman filter that doesn’t take into account correlation of state noise (acceleration) and measurement noise.

***Here is the recommended procedure for part I:*  
*Divergence of Kalman filter when bias of acceleration (state noise) is neglected in assimilation algorithm. Development of optimal Kalman filter that takes into account bias of acceleration (state noise).***

1. Generate a true trajectory of an object motion disturbed by normally distributed **BIASED** random acceleration. Bias (mathematical expectation) of random noise

Size of trajectory is 200 points.

Initial conditions:

Variance of noise ,

1. Generate measurements of the coordinate

–normally distributed random noise with zero mathematical expectation   
and variance .

1. Obtain estimates of state vector by Kalman filter in assumption of unbiased acceleration (). Please use already developed code or detailed recommendations how to develop Kalman filter algorithm in Lab 5. In this conditions it will be non-optimal filter.
2. Plot results including true trajectory, measurements, filtered estimates of state vector .
3. Make runs of filter and estimate dynamics of mean-squared error of estimation over observation interval. Please calculate this error only for filtered estimate of coordinate .

*Hint how to do:*

Calculate squared deviation of true coordinate from its estimation for every run over the whole observation interval =200.

– number of run;

- observation interval

(please start error calculation from step );

- number of runs;

Find average value of over runs for every step and calculate its square root

This final error is true error of estimation as you used true trajectory to get this error.

1. Compare true estimation error obtained in item 5 with errors of estimation provided by Kalman filter algorithm

*Hint how to do:*Make a plot of two curves:

1. Final error (true estimation error) obtained over runs according to item 5.
2. Filtration error covariance matrix (calculation error provided by Kalman filter)  
   Please use square root of the first diagonal element of that corresponds to standard deviation of estimation error of coordinate . It doesn’t depend on runs, for every run it is the same, as it depends only on model parameters

Verify if calculation errors of estimation correspond to true estimation errors.

As bias of acceleration is neglected in Kalman filter algorithm, than true estimation errors should be significantly greater than calculation errors of estimation as filter is non-optimal.

1. Develop optimal Kalman filter algorithm that takes into account bias of acceleration (state noise).

*Hint*

Adjust equation to get the predicted (extrapolated) estimate by introducing correction .

1. Calculate again true estimation errors and errors of estimation provided by optimal Kalman filter that takes into account bias of acceleration (state noise). Compare these errors in optimal conditions.

***Here is the recommended procedure for part II:***

**Sensitivity of estimation results obtained by a Kalman filter that doesn’t take into account correlation of state noise (acceleration) and measurement noise.**

1. Generate a true trajectory of an object motion disturbed by a correlated in time random acceleration. Let’s assume that this random acceleration is first-order Gauss-Markov process. It means that

- uncorrelated random noise with variance );

– variance of correlated noise

– value that is inverse to correlation interval.

For example,

1. if , then and is uncorrelated noise

(substitute 1000 to equation (1) for )

1. if , then is correlated noise on interval over 10 steps. It means that inside every 10 steps correlation is significant.

*Hint to generate correlated noise*

– random uncorrelated unbiased noise with variance .

–standard deviation of random acceleration.

– time interval between measurements.

Then true trajectory is generated using random acceleration obtained according to equation (1). First use that means that is uncorrelated noise,

Size of trajectory is points.

Initial conditions:

1. Generate measurements of the coordinate

– is random correlated noise. This is also first-order Gauss-Markov process.

- uncorrelated random noise with variance );

– variance of measurement noise.

First use that means that is uncorrelated noise.

1. Obtain estimates of state vector by Kalman filter over runs and compare true estimation errors with errors of estimation provided by Kalman filter algorithm. This is optimal Kalman filter as assumptions about uncorrelated noise are true.
2. Generate trajectory and measurements again (repeat items 1 and 2), but use to generate random acceleration according to Equation (1). It means that is correlated noise on interval over 10 steps.
3. Obtain estimates of state vector by Kalman filter over runs in these new conditions. Compare true estimation errors with errors of estimation provided by Kalman filter algorithm. This is non-optimal Kalman filter as assumptions about uncorrelated state noise are not true. Random acceleration is correlated noise.
4. Compare results for optimal and non-optimal filter.
5. Generate trajectory and measurements again (repeat items 1 and 2), but use to generate random acceleration according to Equation (1) ( is uncorrelated noise). But use to generate random measurement noise according to Equation (2). It means that noise is correlated on interval over 10 steps.
6. Obtain estimates of state vector by Kalman filter over runs in these new conditions. Compare true estimation errors with errors of estimation provided by Kalman filter algorithm. This is also non-optimal Kalman filter as assumptions about uncorrelated measurement noise are not true. - is correlated noise.
7. Conclude neglecting of which noise leads to greater accuracy decrease?

***Performance report***

1. Performance report should contain all the items listed
2. The code should be commented. It should include:

* Title of the laboratory work, for example

% Converting a physical distance to a grid distance using least-square method

* The names of a team, indication of Skoltech, and date, for example,   
  %Tatiana Podladchikova, Skoltech, 2016

Main procedures also should be commented, for example

%13-month running mean

…here comes the code

1. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.